Semi-active damping of vibrations.
Prestress Accumulation-Release strategy development.

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Abstract

New method for semi-active control of vibrating structures is introduced. So-called Prestress Accumulation-Release (PAR) strategy aims at releasing of the strain energy accumulated in the structure during its deformation process. Numerical simulations as well as some experimental results prove that the strategy can be very effective in mitigating of the fundamental mode of the structure. An example of a simply supported beam indicates that ca 95% of the fundamental mode of vibration can be mitigated after two cycles. In much more complex practical problems smaller portion of total energy can be released from the system in each cycle, nevertheless the strategy could be applied to mitigate the vibrations of, for example, pipeline systems or pedestrian walkways.

1 Introduction

Prestress Accumulation-Release (PAR) is a new method to convert strain energy of a vibrating system into kinetic energy which is then released from the system by means of a dissipative device. The process is fully semi-active – it does not require adding any substantial amount of energy into the system.

The process consists of two phases. In the first phase some kinematic constraints imposed on the system are released at the instant when the maximum strain energy can be converted to kinetic energy. It is usually manifested with local, higher frequency vibrations. In the second phase kinematic constraints are re-imposed which leads to conversion of kinetic energy in part into other, non-mechanical form, for example heating-up of the actuator device.

First, the proposed approach is described theoretically on a simple spring–mass system in order to demonstrate the idea of response mitigation and show the energy balance of the system.

Secondly the numerical studies are presented for a layered beam simulating a pedestrian bridge, where the control is based on disconnecting for a very short instant of time and sticking back two layers (delamination effect).

Finally, the experimental results are presented. A laboratory scale set up was built to verify the effectiveness of the PAR strategy on a cantilever beam demonstrator. Controllable delamination effect was obtained by means of piezo-electric actuators. The control was carried out as a closed loop feed – back system.

2 Mass – spring system

2.1 The concept

A simple mass – two-springs system is considered as shown on Fig.1a. One of the springs is active in such a sense that it can be detached/ reattached to the mass anywhere along spring length. During the free vibration of the system the active spring can be detached, in particular at the point of maximum displacement of the mass and re-attached as it comes to its free length (cf. Fig. 1b). In the following phase of vibration a force that opposes the further movement is introduced, proportional to the displacement of the active spring from its new equilibrium position. Thus, a new equilibrium of the whole system is established (dotted line on Fig. 1c). Then active spring can be detached/ reattached again resulting in returning to the initial configuration. During following vibration system behaves same as before introducing control, however a considerable part of the total vibration amplitude is diminished.

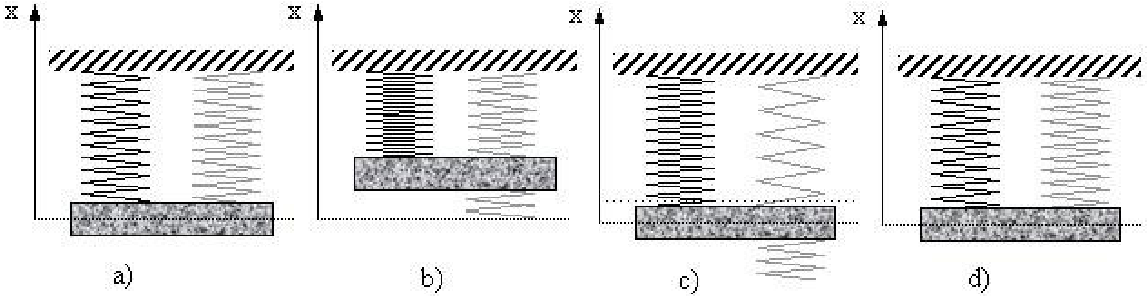


Figure 1: Mass – two-springs system

2.2 Analytical solution

2.2.1 Equation of motion

In this section the system shown on Figure 1 is analyzed in detail. If natural damping is not considered and no force excitation is used, then the motion in the first phase of vibration is governed by the equation:

$$m\ddot{x}(t) + (k_1 + k_2) \cdot x(t) = 0 \tag{1}$$

where m is the moving mass and $k_i = \dfrac{E_i \cdot A_i}{L_i}$ is the stiffness of a spring with cross section A_i, Young Modulus E_i and length L_i. The solution, given the initial conditions $x(t=0) = -\varepsilon$, and $\dot{x}(t=0) = 0$, takes the form:

$$x(t) = -\varepsilon \cdot \cos\left(\sqrt{\frac{k_1 + k_2}{m}} t\right) \tag{2}$$

At the time instant of maximum displacement, t = t^1 the active spring is detached and reattached as it comes to its equilibrium position (cf. Fig. 1b). At this point it is assumed that the inertia of springs is not taken into account. Now the equation of motion is given by:

$$m\ddot{x}(t) + k_1 \cdot x(t) + k_2^{'} \cdot x_2(t) = 0 \tag{3}$$

where $k_2^{'} = \frac{E_2 \cdot A_2}{L_2^{'}}$ with new active spring length $L_2^{'} = L_2 - \varepsilon$, and x$_2$ is the active spring displacement in the second phase: $x_2(t) = x(t) - \varepsilon$. The system has still a single degree of freedom, namely the displacement of the mass *m*. Equation (3) can be rewritten as:

$$m\ddot{x}(t) + (k_1 + k_2^{'}) \cdot x(t) = k_2^{'} \cdot \varepsilon \tag{4}$$

It can be seen that in the second phase of process the governing equation is non-homogenous with a term $k_2^{'} \cdot \varepsilon$, which can be viewed as an additional, constant force applied to the system. Now the solution takes the form:

$$x(t) = C_1 \cdot \cos\left(\sqrt{\frac{k_1 + k_2^{'}}{m}}t\right) + C_2 \cdot \sin\left(\sqrt{\frac{k_1 + k_2^{'}}{m}}t\right) + \frac{k_2^{'} \cdot \varepsilon}{k_1 + k_2^{'}} \tag{5}$$

with constants C$_1$ and C$_2$ calculated from initial conditions: $x(t = t^1) = \varepsilon$, and $\dot{x}(t = t^1) = 0$.

This operation can be repeated several times in order to enhance the amplitude mitigation effect.

If the desired effectiveness is reached than the operation of detaching/ reattaching can be repeated again near the initial equilibrium position in order for the active spring to return to its initial length.

2.2.2 Energy balance

Potential energy of a spring is equal to the work of the elastic force done along the displacement direction:

$$E_{POT} = \int_0^{x_k} k_i \cdot x dx = \frac{1}{2} \cdot k_i \cdot x_k^2 \tag{6}$$

Potential energy of the system in the instant before activation of control (t = t^1 - dt) is:

$$E_{POT} = \frac{1}{2} k_1 \cdot \varepsilon^2 + \frac{1}{2} k_2 \cdot \varepsilon^2 \tag{7}$$

Potential energy of the system in the instant after activation of control (t = t^1 + dt) is:

$$E_{POT} = \frac{1}{2} k_1 \cdot \varepsilon^2 + \frac{1}{2} k_2^{'} \cdot (\varepsilon - \varepsilon)^2 = \frac{1}{2} k_1 \cdot \varepsilon^2 \tag{8}$$

The control is activated at the point of maximum displacement, where $\dot{x} = 0$, thus at this point the kinetic energy vanishes. If inertia of springs was not neglected then some energy would be transferred to this part of active spring which can vibrate freely after reattachment. This situation is discussed in further sections.

2.2.3 Numerical example

Figure 2 depicts the resulting displacement of the mass if the following data were used for calculations:

- m = 20 kgs
- L = 0.1 m
- E = 6e10 Pa
- A = 1.54e-8 m^2
- **ε** = 0.01 m

$\left.\begin{array}{l} L \\ E \\ A \end{array}\right\}$ both springs equal

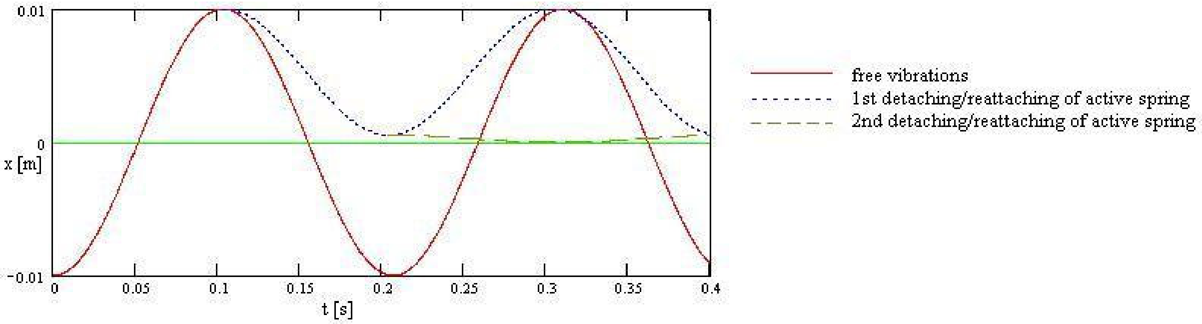


Figure 2: Resulting mass displacement; control triggered twice

After performing detaching/reattaching of the active spring twice vibrations of the mass ceased almost completely. Slight change in frequency of controlled response as compared with reference case is due to small change of spring stiffness which affects the frequency and is caused by change of spring length. It can also be observed that after activating the control the system oscillates about new equilibrium position (cf. dotted line on Fig.2).

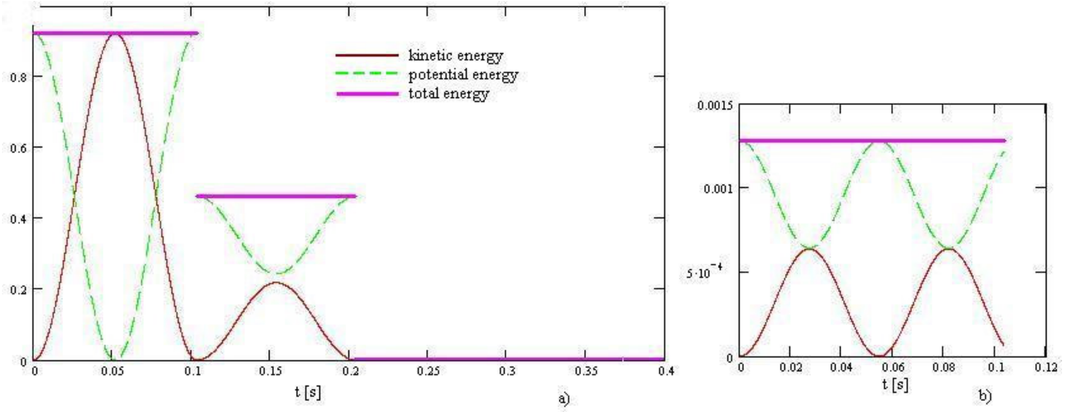


Figure 3: Energy balance

Activating control takes place at the maximum value of accumulated strain energy and results in instant decrease of strain energy (cf. Fig. 3, Eq. 7 and Eq. 8). At the end of the process almost all energy was released from the system. In more practical approach part of the energy would be transferred into higher frequency vibrations of the detached part of active spring, and part would be dissipated in the process which is here idealized as imposing some initial values. The last phase of the process is zoomed on Figure 3b).

2.3 Case with inertia of active spring considered

2.3.1 Introduction

As said before, detaching of the active spring results in converting accumulated strain energy into kinetic energy that can be dissipated from the system during reattaching of the spring. This whole process was idealized in the previous section with imposing proper initial conditions which resulted in instant decrease in energy of the system. In practice part of the released strain energy is dissipated by a device which reattaches the spring and the remaining part introduces higher frequency vibrations which can be, however easily suppressed with natural damping of the system.

In present analysis control device is idealized with imposing/releasing of local constraints between geometrical point of mass *m* and any point along the active spring. The mass of active spring is concentrated at its full length and in the middle (cf. Fig 4).

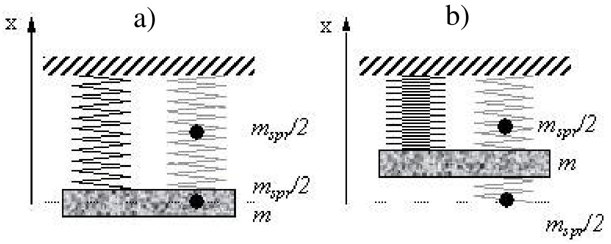
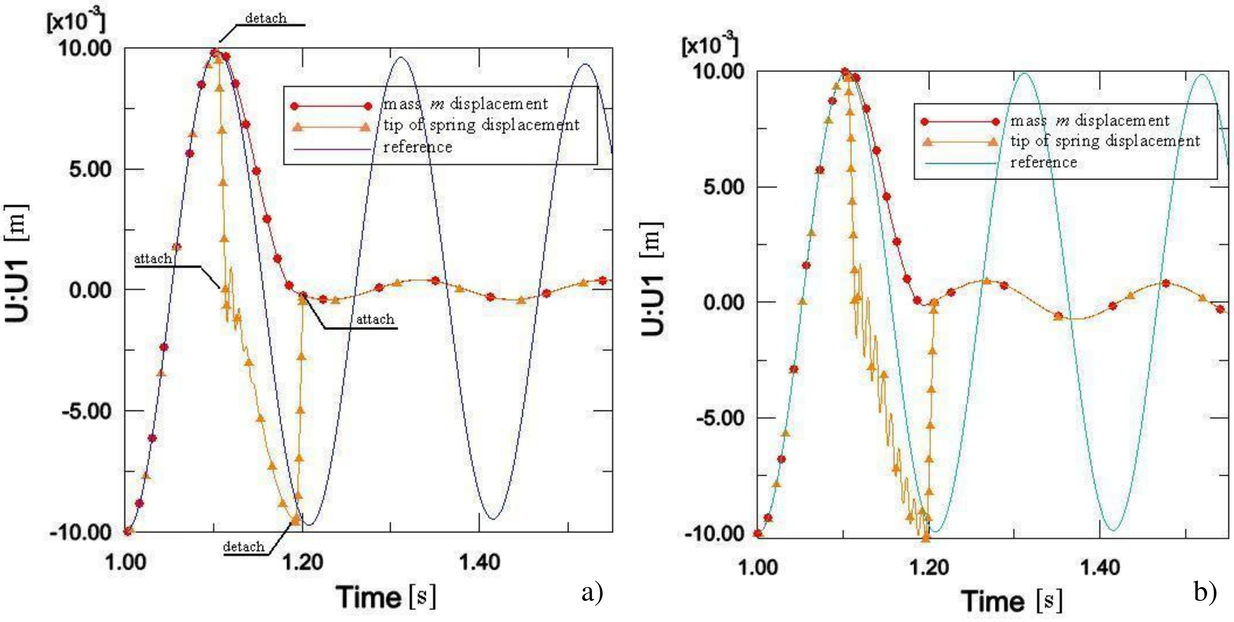


Figure 4: Dynamic model with inertia of active spring included

Slight natural damping is also introduced into the equations. Chosen Rayleigh damping coefficients introduce slight damping (1% or 0.2%) around first natural frequency and relatively much higher damping of higher frequencies. All remaining parameters do not change. Simulations were performed using Abaqus/Standard code.

2.3.2 Results

Displacements of mass *m* and tip of the spring are depicted on Figure 5. Detaching of active spring and reattaching it at some point along its length introduces higher frequency vibrations of the spring end. Releasing/ re-imposing the constrain again results in returning of the system to initial configuration, but with the amplitude of vibrations decreased.



a) b)

Figure 5: Response of the system; a) 1% of critical damping b) 0.2% of critical damping

First detaching of spring is done at the point of maximum displacement, i.e. where there is maximum stress accumulated in the spring. Then the spring is reattached when its end passes the equilibrium position which means that the length of spring is decreased by about 10 mm. This corresponds to Figure

4b). If the control was stopped here, than the system would oscillate about a new equilibrium position. The time instant for the next detaching of spring has to be chosen properly, so that it can be reattached as the mass *m* is as close to the initial equilibrium position as possible. The whole procedure can be repeated several times, if needed. If, for instance, active spring stiffness was considerably smaller then passive one, then desired mitigation effect would have to be obtained in more steps. In the analyzed example after the described sequence of activating the control amplitude of mass *m* displacement was decreased by 96% and 92% for 1% and 0.2% of critical damping, respectively. The procedure is very sensitive to the time instant of detaching the spring.

Re-imposing the constrains causes higher frequency vibrations of the mass located in the middle of the spring. These vibrations however are effectively damped out by the natural damping of the system. Typical behavior of middle mass is shown on Figure 6.

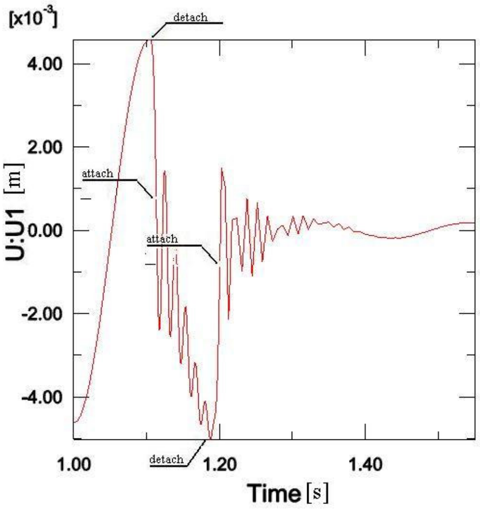
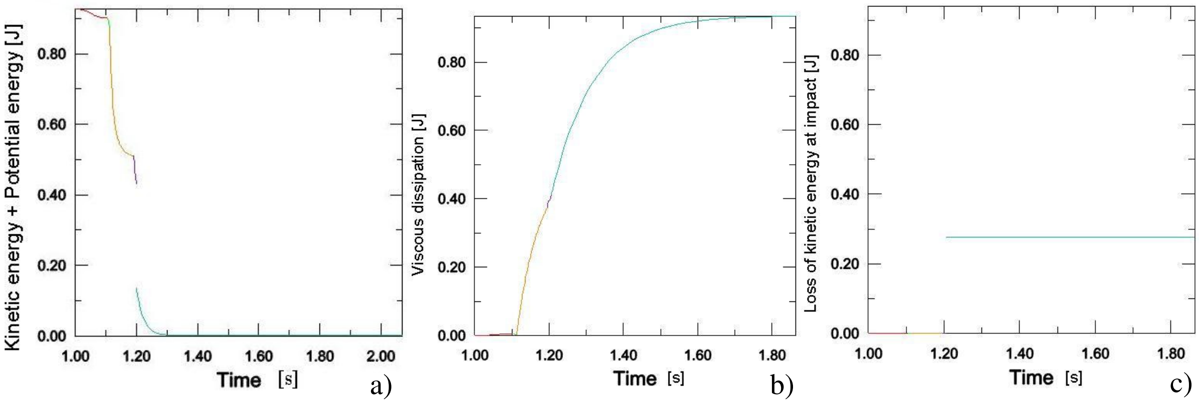


Figure 6: Middle mass response with indicated points corresponding to activating control

Steep, exponential decline in the sum of potential and kinetic energy graph is due to viscous dissipation which increases with increase of vibration velocity. Vibration velocity, in turn, is highest as the local, higher frequency vibrations are introduced due to imposing/ releasing constrains. The viscous dissipation is due to natural damping of the system.

Discontinuity of the graph is caused by the loss of kinetic energy at the instant when constrains are re-imposed. The size of this gap indicates the maximum amount of energy that can be dissipated by the active device. All non-zero forms of energy can be viewed on Figure 7.



a) b) c)

Figure 7: a) sum of strain and kinetic energy b) viscous dissipation due to natural damping c) loss of kinetic energy during imposing constrains

3 Delamination of layered beam

3.1 PAR strategy for layered beams

The strategy of releasing the accumulated strain energy in order to dissipate it can be in theory effectively used for various types of structures. If a layered beam is considered as shown on Figure 8, the idea of adaptation would be as follows. First, at the point of maximum deflection two layers are disconnected resulting in almost instant dislocation of layers (1' on Fig. 8a). This dislocation can than be frozen if the layers are reconnected again. This yields introduction of elastic force that opposes the further vibration of the cantilever (2 on Fig. 8a). Then, near the equilibrium position layers are disconnected/ reconnected again in order to return to initial configuration. The whole sequence can be repeated until the desired effect is obtained.

Similar effect of response mitigation can be obtained if a truss structure is considered with a detachable element (cf Fig. 8b). Applying the same methodology for control, the axial strain accumulated in the active element can be released as the element is disconnected at one of its ends.

It is worth mentioning that in both cases only one active member is required.

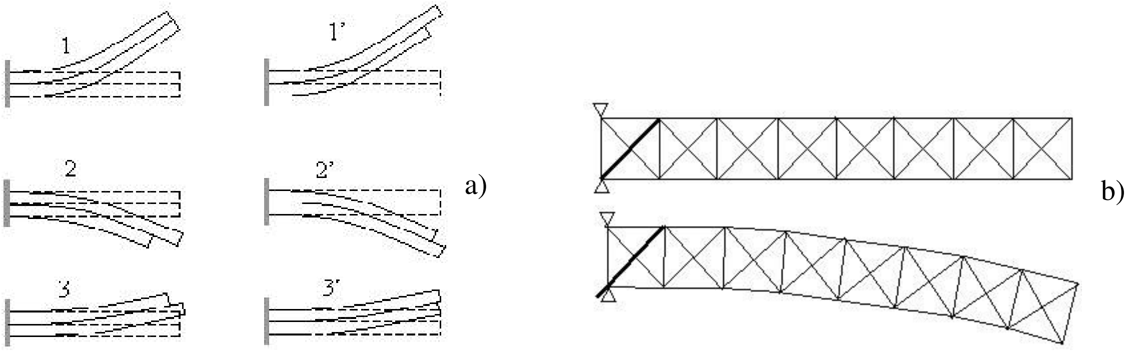


Figure 8: PAR strategy for a cantilever beam

3.2 Numerical example – simply supported beam

3.2.1 Numerical model

A simply supported, two-layered beam with the span of 15.6 m is considered. Bending stiffness of each layer is EI = 2.218 x 10^6 Nm^2. Material damping of 1% of critical damping is assumed around 1st natural frequency. Layers are permanently connected together at the left support. It is assumed that there is a device at the right support capable of instant disconnecting or sticking the layers. Along the beam length the distance between layers remains the same, whereas the frictionless, relative movement of layers is possible in the direction parallel to the beam axis. Considered beam model is depicted on Figure 9.

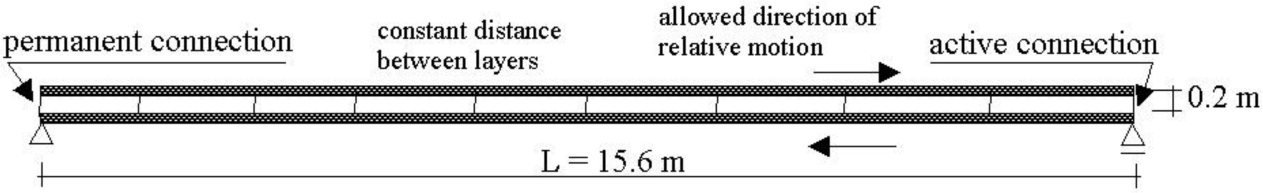


Figure 9: Assumed model of layered beam

3.2.2 Results

In the first step an initial displacement of 16 cm was applied to the model. In the following steps dynamic analysis procedure was used to calculate the free vibrations of the system as the reference case. Then calculations were repeated with control procedures added. The vertical displacement of middle of span is shown on Figure 10.

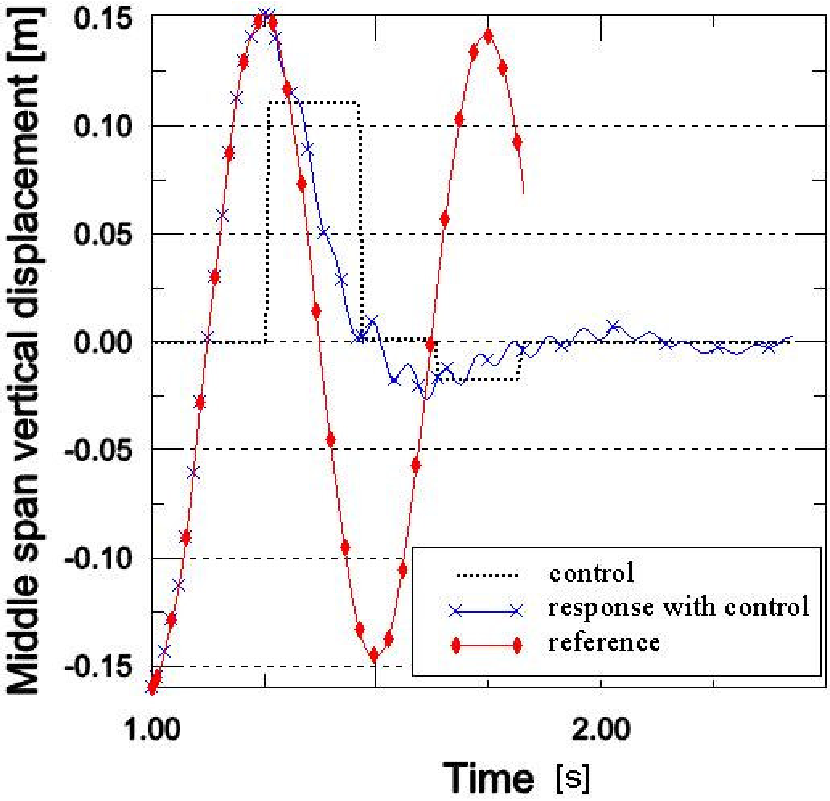


Figure 10: Vertical displacement of the middle of beam

It can be observed that ca. 95% of the vibration amplitude is damped out after two cycles of vibration. The control, i.e. the relative displacement between layers' ends, shown on the picture is magnified 10 times. After only two cycles of disconnection/ reconnection of layers the vibration of the first mode is considerably mitigated, while higher modes vibrations are introduced. The second sequence of control activation is triggered close to the maximum accumulated strain, at point where the deformation shape is of the first mode (cf. Fig. 11a). It is worth mentioning that if the control was triggered at the local peak, short after the first sequence it would not give any desired effect since the deformation shape at that point is of third mode of vibration (cf. Fig. 11b). The performed control does not affect higher modes which can be however mitigated with the natural damping. Appropriate modification of control strategy could also make PAR a useful tool in higher modes mitigation.

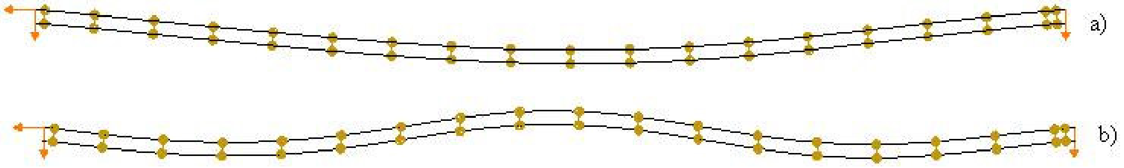


Figure 11: desired a) and undesired b) deformation shape for activating the control

The control in the numerical simulations was applied by instant releasing the available component of relative motion between layers or freezing it at some value. It resulted in very fast dislocation of layers in direction parallel to the beam axis. Of course, a drawback of such approach is that between subsequent disconnecting and reconnecting of layers the beam stiffness is decreased. However the time intervals when this is the case is very short as can be seen on Figure 12. It can also be observed that at the end of the process relative displacement between layers is zero which means that the structure returned to the initial configuration.

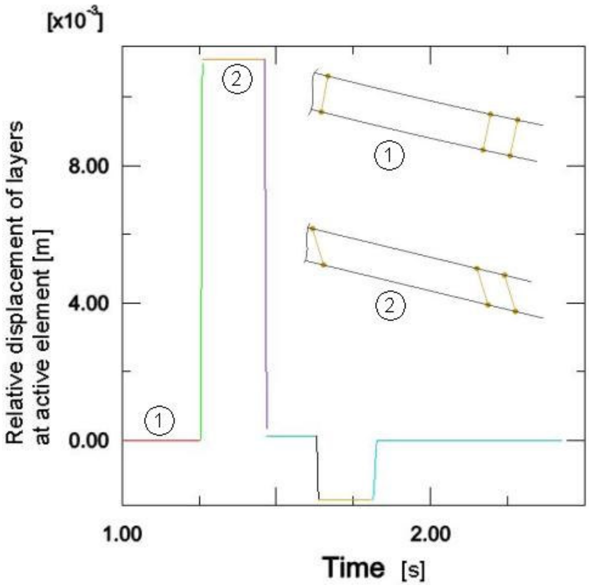


Figure 12: Relative displacement of layers

4 Experimental set-up

Experimental verification of the influence of the controlled *delamination effect* on damping of free vibration has been performed using the double layer beam model (cf. Fig.13 and Table 1) equipped with actively controlled piezoelectric clamping device. The clamper can realize two functions: full adhesion between the two-beam layers and full *delamination*. The active control process is reduced to the strategy of the clamper switching on and off. The time intervals for clamper opening have been determined experimentally, as the smallest interval required releasing full *delamination* effect.

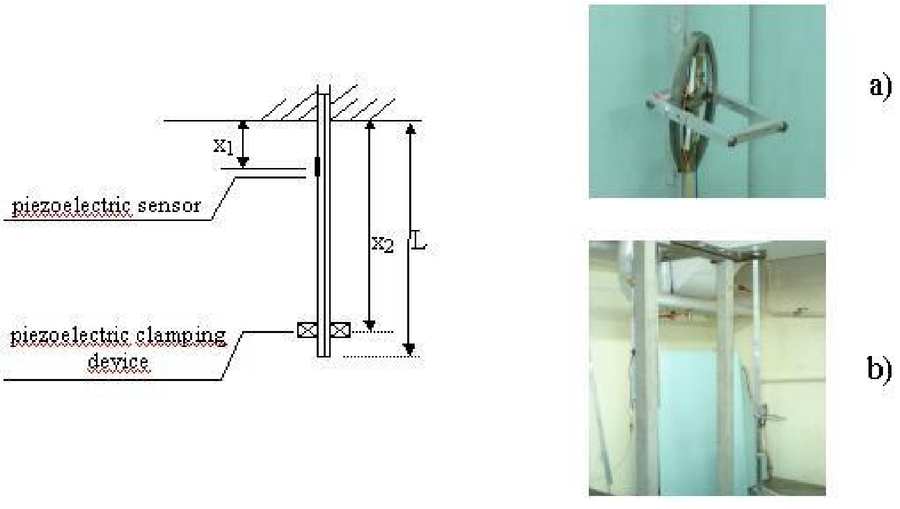


Figure 13: Experimental set-up; a) clamping device b) general view

L = 1.34 [m]	mass of sticker M = 0.3 [kg]
$x_1 = 2.7$ [cm]	Sticking force ≈ 500 [N]
$x_2 = 1.30$ [m]	Excitation by initial deflection at tip point

Table 1: Technical data for the experimental set-up

The experimental results for the actively controlled beam have been shown on Figure 14, where oscillating lines correspond to signal obtained from strain sensor glued to the beam surface. The control strategy for the sticker operation (locking/unlocking) is shown through the step function. Despite of high natural damping of the tested model, it the efficiency of the applied active damping strategy can be seen. As mentioned before the strategy is quite sensitive to the choice of time instants of locking/unlocking. Therefore optimal tuning of the control is believed to much enhance the results as compared with shown initial tests.

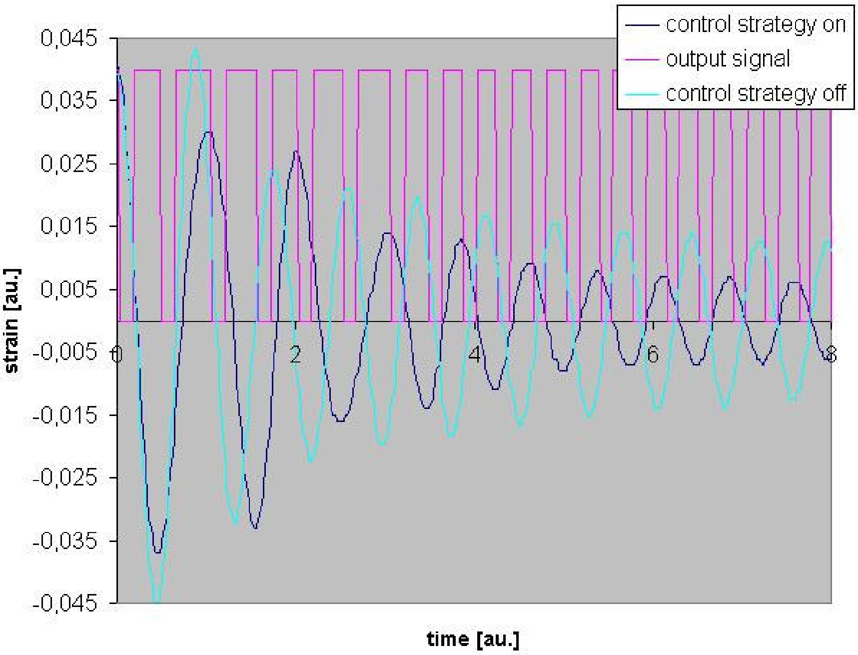


Figure 14: Actively damped vibration (the first mode) of the double layer beam and the applied control strategy

5 Possible applications

Among possible applications are:

- Pedestrian bridges
- Pipelines
- Truss structures

Two layered beam discussed in section 3 has the span and stiffness of an experimental, light weight pedestrian bridge located in EMPA Laboratory, Switzerland. Cables sustaining the span, which completely change the dynamic behavior of the structure were not modeled in the numerical example. In order to effectively apply PAR strategy to this type of structures, mitigation of higher modes has to be accounted for. For this purpose only one active member at the support is not sufficient.

Detachable spring which was discussed in section 2 could be used at the supports of pipeline systems in order to accumulate the deformation energy and convert it to kinetic energy which can be than dissipated.

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- **SAFE-STRUCT** *New methods for design of safe structures with identification of hazards and active adaptation in critical states.*

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